EXPONENTIALS AND LOGARITHMS

1 Given that $a = \log_{10} 2$ and $b = \log_{10} 3$, find expressions in terms of a and b for

a $\log_{10} 1.5$, **(2)**

b $\log_{10} 24$, **(2)**

c $\log_{10} 150$. **(3)**

2 Find, to an appropriate degree of accuracy, the values of x for which

a $4 \log_3 x - 5 = 0$, **(2)**

b $\log_3 x^3 - 5 \log_3 x = 4$. **(3)**

3 **a** Given that $p = \log_2 q$, find expressions in terms of p for

i $\log_2 \sqrt{q}$,

ii $\log_2 8q$. **(4)**

b Solve the equation

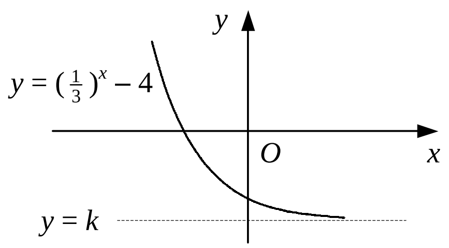
$$\log_2 8q - \log_2 \sqrt{q} = \log_3 9.$$ **(3)**

4 An initial investment of £1000 is placed into a savings account that offers 2.2% interest every 3 months. The amount of money in the account, £P, at the end of t years is given by

$$P = 1000 \times 1.022^{4t}$$

Find, to the nearest year, how long it will take for the investment to double in value. **(4)**

5



The diagram shows the curve with equation $y = (\frac{1}{3})^x - 4$.

a Write down the coordinates of the point where the curve crosses the y-axis. **(1)**

The curve has an asymptote with equation $y = k$.

b Write down the value of the constant k. **(1)**

c Find the x-coordinate of the point where the curve crosses the x-axis. **(3)**

6 **a** Solve the equation

$$\log_3 (x + 1) - \log_3 (x - 2) = 1.$$ **(3)**

b Find, in terms of logarithms to the base 10, the exact value of x such that

$$3^{2x+1} = 2^{x-4}.$$ **(3)**

7 **a** Given that $t = 2^x$, write down expressions in terms of t for

i 2^{x-1},

ii 2^{2x+1}. **(3)**

b Hence solve the equation

$$2^{2x+1} - 14(2^{x-1}) + 6 = 0.$$ **(5)**

8 Find the values of x for which

a $\log_2 (3x + 5) + \log_5 125 = 7$, **(3)**

b $\log_2 (x + 1) = 5 - \log_2 (3x - 1)$. **(5)**

9 Given that $\log_a (x + 4) = \log_a \frac{x}{4} + \log_a 5$,

and that $\log_a (y + 2) = \log_a 12 - \log_a (y + 1)$,

where $y > 0$, find

a the value of x, **(3)**

b the value of y, **(4)**

c the value of the logarithm of x to the base y. **(2)**

10 A colony of fast-breeding fish is introduced into a large, newly-built pond. The number of fish in the pond, n, after t weeks is modelled by

$$n = \frac{18000}{1 + 8c^{-t}}.$$

a Find the initial number of fish in the pond. **(2)**

Given that there are 3600 fish in the pond after 3 weeks, use this model to

b show that $c = \sqrt[3]{2}$, **(3)**

c find the time taken for the initial population of fish to double in size, giving your answer to the nearest day. **(4)**

11 **a** Given that $y = \log_8 x$, find expressions in terms of y for

i $\log_8 x^2$,

ii $\log_2 x$. **(4)**

b Hence, or otherwise, find the value of x such that

$$3 \log_8 x^2 + \log_2 x = 6.$$ **(3)**

12 Solve the simultaneous equations

$$\log_2 y = \log_2 (3 - 2x) + 1$$

$$\log_4 x + \log_4 y = \tfrac{1}{2}$$ **(8)**

13 **a** Sketch on the same diagram the curves $y = 2^x + 1$ and $y = (\frac{1}{2})^x$, showing the coordinates of any points where each curve meets the coordinate axes. **(4)**

Given that the curves $y = 2^x + 1$ and $y = (\frac{1}{2})^x$ intersect at the point A,

b show that the x-coordinate of A is a solution of the equation

$$2^{2x} + 2^x - 1 = 0,$$ **(2)**

c hence, show that the y-coordinate of A is $\frac{1}{2}(\sqrt{5} + 1)$. **(4)**

14 **a** Show that $x = 1$ is a solution of the equation

$$2^{3x} - 4(2^{2x}) + 2^x + 6 = 0. \qquad \text{(I)}$$ **(1)**

b Show that using the substitution $u = 2^x$, equation (I) can be written as

$$u^3 - 4u^2 + u + 6 = 0.$$ **(2)**

c Hence find the other real solution of equation (I) correct to 3 significant figures. **(7)**